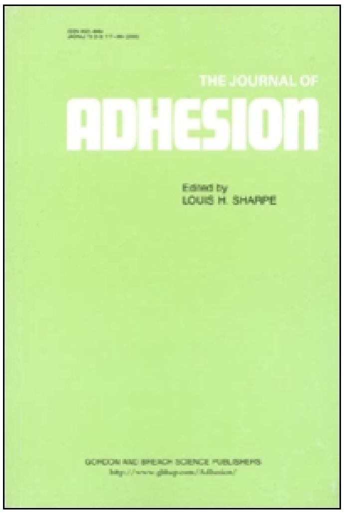

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Fracture Mechanics of a Shaft-loaded Blister Test – Transition from a Bending Plate to a Stretching Membrane

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A linear elastic solution is proposed for a circular disc in transition from a plate-like (pure bending) to a membrane-like behavior (pure stretching) under a central point load. The strain energy release rate for film delamination is found to be $G = \chi(Pw_0/\pi a^2)$ with χ a numerical constant varying from 1/2 for a plate-like disc to 1/4 for a thin flexible membrane.

Keywords: Blister test; circular plates; delamination; fracture; bending; stretching; shaft-loaded blister

1. INTRODUCTION

Thin film adhesion can be measured directly by a one-dimensional V-peel test [1, 2] or an axisymmetrical blister test [3, 4]. In a V-peel test, an external load is applied directly to a rectangular film with two opposite ends attached to a substrate *via* a horizontal cylinder. The fracture mechanics and transition from a thick bending plate to a thin stretching film was discussed in an earlier paper [1]. In this paper, we will focus on the shaft-loaded blister test where a central point load is to drive a blister delamination. Transforming the V-peel formulation into a cylindrical coordinate system, an elastic solution is computed

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for an elastically-deformed clamped plate of varying film thickness and bending-to-stretching ratio. A mechanical energy release rate will also be calculated to account for the delamination process.

2. ELASTIC RESPONSE WITHOUT DELAMINATION

For a circular plate of flexural rigidity $D = Eh^3/12(1-\nu^2)$, elastic modulus, E, Poisson ratio, ν, radius, a, and thickness, h, a central point load, F, is applied to create a blister deflection, $w(r)$, with a central defection, w_0 (Fig. 1). The plate is elastically deformed under a mixed mode of bending and stretching. The membrane stress can be approximated by a polynomial expansion [5, 6], but we will adopt an average membrane stress of N, equal to both the radial and tangential stresses, so that $N_r = N_t = N$. At equilibrium, linear elasticity requires [7]

$$\frac{d}{dr}(N_r + N_t) + \frac{Eh}{2r}\left(\frac{dw}{dr}\right)^2 = 0, \tag{1}$$

$$Q_r + N_r\frac{dw}{dr} + \frac{F}{2\pi r} = 0 \tag{2}$$

where the shear force is given by

$$Q_r = -D\left(\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr}\right). \tag{3}$$

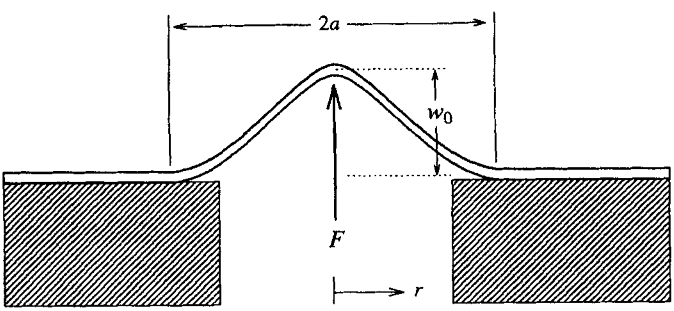

FIGURE 1 Schematic of a shaft-loaded blister test.

Substituting (3) into (2) and combining with (1), we get

$$\frac{d^3w}{dr^3}+\frac{1}{r}\frac{d^2w}{dr^2}-\frac{1}{r^2}\frac{dw}{dr}-\frac{N}{D}\frac{dw}{dr}=\frac{F}{2\pi Dr}. \tag{4}$$

A few useful dimensionless parameters are defined as:

$$\xi=\frac{r}{a},\quad W=\frac{w}{h},$$

$$\theta=\frac{dW}{d\xi}=\frac{a}{h}\frac{dw}{dr},\quad \theta'=\frac{d\theta}{d\xi},$$

$$\beta=a\left(\frac{N}{D}\right)^{1/2},\quad \varphi=\frac{Fa^2}{2\pi Dh}.$$

The normalized membrane stress, β, indicates the ratio of stretching stress to bending rigidity. A small β represents a pure bending plate, while a large β indicates a stretching membrane. Residual stress can be conveniently accounted for by adding an extra term $\sigma_{\mathrm{res}}h$ to N, but will not be discussed in this paper. Rewriting (4) in terms of the dimensionless quantities, we get

$$\xi^2\frac{d^2\theta}{d\xi^2}+\xi\frac{d\theta}{d\xi}-(1+\beta^2\xi^2)\theta=\xi\varphi. \tag{5}$$

This modified Bessel equation can be solved exactly[1] yielding

$$\theta=C_1I_1(\beta\xi)+C_2K_1(\beta\xi)-\frac{\varphi}{\beta^2\xi} \tag{6a}$$

where I_1 and K_1 are the modified Bessel functions of the first and second kind, respectively, and C_1 and C_2 are constants. There are three boundary conditions, namely, (i) $\theta|_{\xi=1}=0$, (ii) $\theta|_{\xi=0}=0$ and (iii) $W|_{\xi=1}=0$. To satisfy (i) and (ii), $C_1=(\varphi/\beta^2)[1-\beta K_1(\beta)]/I_1(\beta)$, $C_2=\varphi/\beta$, and (6a) becomes

$$\theta=\frac{\varphi}{\beta^2}\left[\left(\frac{1-\beta K_1(\beta)}{I_1(\beta)}\right)I_1(\beta\xi)+\beta K_1(\beta\xi)-\frac{1}{\xi}\right]. \tag{6}$$

[1]Note that if N is a polynomial in r rather than a constant, (5) becomes non-linear and does not yield an analytical solution.

The blister profile can be found by integrating θ with respect to ξ from 0 to 1, and matching boundary condition (iii), or,

$$W = \frac{\varphi}{\beta^2}\left\{\left[\frac{1-\beta K_1(\beta)}{\beta I_1(\beta)}\right][I_0(\beta\xi) - I_0(\beta)] - [K_0(\beta\xi) - K_0(\beta)] - \log\xi\right\} \tag{7}$$

with a central deflection[2]

$$W_0 = \frac{\varphi}{\beta^2}\left\{\left[\frac{1-\beta K_1(\beta)}{\beta I_1(\beta)}\right][1 - I_0(\beta)] - \log\left(\frac{2}{\beta}\right) + \gamma + K_0(\beta)\right\} \tag{8}$$

where $\gamma = 0.577216$ is the Euler–Mascheroni constant. Figure 2 shows a normalized blister profile W/W_0 for various β. In an ultra-thin film where bending is negligible, a spike is present at the plate center

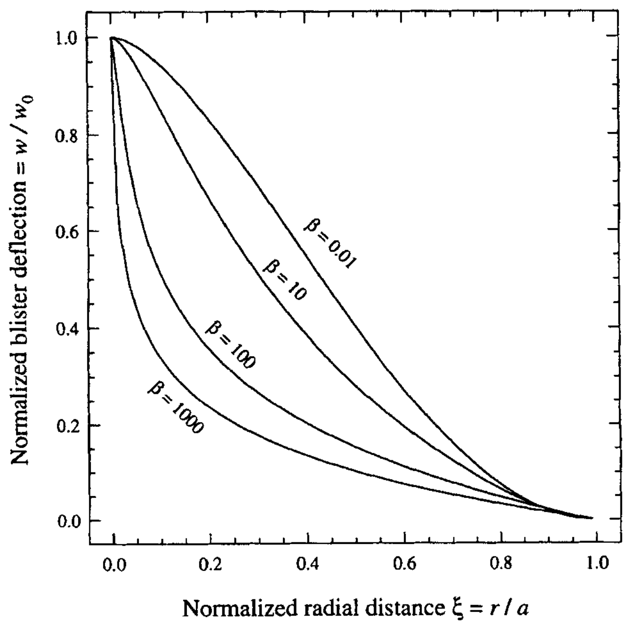

FIGURE 2 Normalized blister profile for $\beta = 10^{-2}$, 10, 10^2 and 10^3.

[2]As $K_0(z) \approx \log(2/z) - \gamma$ and $I_0(z) \approx 1$ for $z \to 0$ [8].

because of the stress concentration. The boundary conditions (i) and (ii) are no longer matched in this stretching limit.

The membrane stress N can be calculated by considering the strain on the film,

$$N = \frac{Eh}{2a^2(1-\nu^2)}\int_0^a (dw/dr)^2 r dr = \frac{Eh^3}{2a^2(1-\nu^2)}\int_0^1 \theta^2 \xi d\xi. \tag{9}$$

Substituting (6) into (15), the normalized applied load becomes

$$\varphi = \frac{\beta^3}{[6f(\beta)]^{1/2}} \tag{10}$$

with

$$\begin{aligned} f(\beta) = {} & \frac{1}{2}\left(1 - \frac{1}{\beta^2}\right) + 2\left[\frac{1-\beta K_1(\beta)}{I_1(\beta)}\right] \\ & + \frac{1}{2}\left\{\left[\frac{1-\beta K_1(\beta)}{I_1(\beta)}\right] I_0(\beta) - \beta K_0(\beta) - \frac{1}{\beta}\right\}^2 \\ & - \frac{2}{\beta}\left\{\left[\frac{1-\beta K_1(\beta)}{I_1(\beta)}\right] I_0(\beta) - \beta K_0(\beta)\right\} - [\log(2) - \gamma]. \end{aligned} \tag{11}$$

By eliminating β from (8) and (10), the elastic response $\varphi(W_0)$ of the circular disc can be found. Figure 3 shows a parametric plot of $\varphi(W_0)$ in a log–log plot with the parameter β varying from 0.01 to 100. The gradient $n = [d(\log\varphi)/d(\log W_0)]$, increases from $n = 1$ in the "bending dominant" region (small W_0) to approximately $n = 3$ in the "stretching dominant" region (large W_0). A pronounced bending-to-stretching transition occurs in the range roughly from $W_0 = 0.1$ (with $n = 1.01$) to 20 (with $n = 2.5$). Such a bending-to-stretching transition is seen also in a pressure *versus* blister height relationship in a pressurized blister [9].

Two limiting cases are noted: (i) the pure bending plate and (ii) the pure stretching membrane. For a plate under pure bending where N is negligible and $\beta \to 0$, (6) to (8) collapse to[3]

$$\theta = \frac{\varphi\xi}{2}\log\xi, \tag{12}$$

[3]As $K_1(z) = (1/z) + [(1/2)\log(z/2) + 2\gamma - 1]z$ [8].

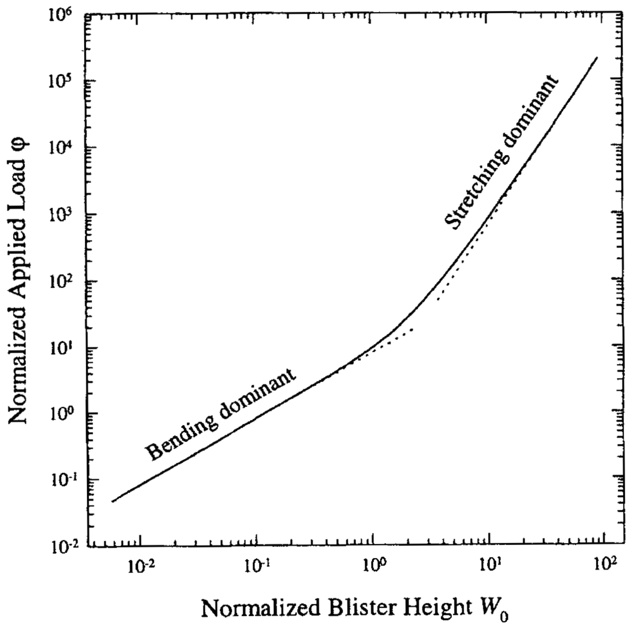

FIGURE 3 Normalized applied load as a function of normalized blister height.

$$W = \frac{\varphi}{8}(1 - \xi^2 + \xi^2 \log \xi^2) \quad \text{or}$$
$$w = w_0 \left[1 - \left(\frac{r}{a}\right)^2 + \left(\frac{r}{a}\right)^2 \log \left(\frac{r}{a}\right)^2 \right] \tag{13}$$

$$W_0 = \frac{\varphi}{8} \quad \text{or} \quad w_0 = \frac{3Fa^2(1-\nu^2)}{4\pi E h^3}. \tag{14}$$

This is the classical solution for a clamped plate [7, 10, 12]. Here $F(w_0)$, or $\varphi(W_0)$, is linear in (14) with $n = 1$ as expected. Note that this elastic solution can also be derived by putting $\beta = 0$ in (4) so that $\xi^2\theta'' + \xi\theta' - \theta = \xi\theta$ and, hence, (12) to (14). For a thin film under pure stretching such that D is negligible and $\beta \to \infty$, (6) and (7) become

$$\theta = -\frac{\varphi}{\beta^2 \xi} \quad (\approx 0), \tag{15}$$

$$W = -\frac{\varphi}{\beta^2}\log\xi \tag{16}$$

respectively. Therefore, (15) becomes

$$F = -2\pi r(Nh)\frac{dw}{dr}, \tag{17}$$

which is a balance of vertical forces acting on the membrane at equilibrium (*cf.* $\sin(dw/dr) \approx (dw/dr)$). This elastic solution was first shown by Williams [11]. Here W_0 is ostensibly unbounded because of the presence of the logarithmic term (*cf.* (16)), but the contribution due to φ keeps W_0 within bound (*cf.* (10)). In fact, $\varphi \propto W_0^3$ (or $F \propto w_0^3$) and $n = 3$, as shown in Figure 3. The cubic relation was demonstrated earlier by Wan and Mai by assuming a straight-sided conic geometry [3]. Note that (15) and (16) can be derived alternatively by putting $D = 0$ so that (4) collapses to $\beta^2\xi^2\theta = \xi\varphi$ and, hence, (15) to (17).

3. FRACTURE MECHANICS OF BLISTER DELAMINATION

For an axisymmetric blister delamination to occur at a fixed F, a mechanical energy release rate can be found by

$$G = \frac{dU_c}{dA}\bigg|_F \tag{18}$$

where U_c is the complimentary energy and $A = \pi a^2$ is the crack area. Since $\varphi \propto A$, a normalized G, defined to be $\chi = G/(Fw_0/A)$ can be written as [1]

$$\chi = \Omega n = \Omega\left[\frac{d(\log\varphi)}{d(\log W_0)}\right] \tag{19}$$

with the normalized complimentary energy $\Omega = \int W_0\, d\varphi/(\varphi W_0)$. Comparing with the V-peel test [1], (19) differs by a factor of 3, since $\varphi_v \propto W_0^3$ (with the subscript v denoting V-peel). Substituting (8) and (10) into (19), $\chi(\beta)$ can be plotted as shown in Figure 4 with $(1/2) \leq \chi \leq (1/4)$. The lower bound is the classical result for a bending plate [10, 12] and the upper bound for a stretching membrane [11].

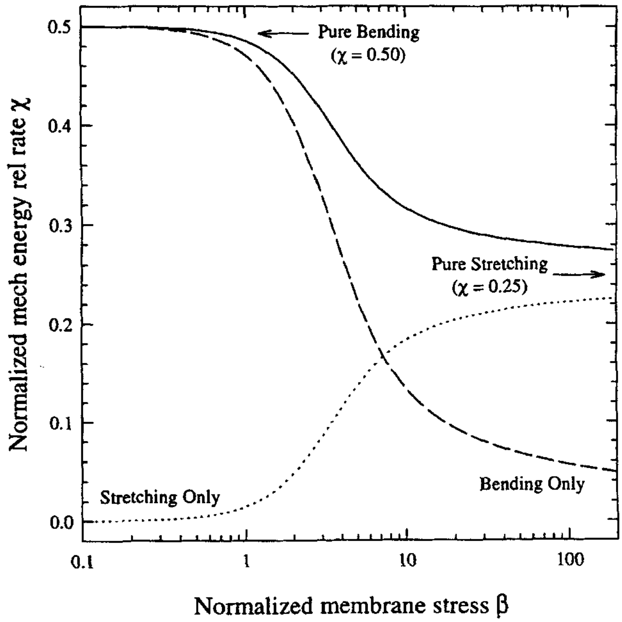

FIGURE 4 Normalized strain energy release rate as a function of β (solid line). Contributions due to bending χ_b (dashed line) and stretching χ_s (dotted line) are also shown.

Note that at $\beta = 100$, when $W_0 \approx 92$, the bending contribution remains significant at roughly 14% and $\chi = 0.28$. The bending-to-stretching transition occurs in the range roughly from $\beta = 0.30$ (with $\chi = 0.4990$) to 100 (with $\chi = 0.2781$).

The stability of a shaft-loaded blister delamination can be determined by investigating the dependence of G upon A. At equilibrium, (18) and (19) yield $G \propto F^{(1+1/n)} A^{(1/n-1)}$. For a pure bending plate ($n = 1$), $(\partial G/\partial A) = 0$ and, therefore, there is a neutral equilibrium [13]. As the blister crack propagates, F remains constant, independent of A. This was demonstrated in both theory and experiment by Malyshev and Salganik [10]. For a pure stretching membrane ($n = 3$), $(\partial G/\partial A) < 0$ and, therefore, there is a stable equilibrium [13]. Here $F \propto A^{1/2}$ [3]. The mechanical behavior of a film under mixed bending/stretching is, therefore, expected to be "stable" in one extreme to "neutral" in another. It is interesting to compare the fracture stability of a shaft-loaded blister test with a V-peel test. The latter shows a theoretical instability in all cases, with the exception of an extremely flexible film which is characterized by a "neutral" crack growth. It is, therefore, expected that an elliptical crack will grow in a stable, neutral and unstable manner depending on the film thickness and its eccentricity, e (where $e = 0$ for an axisymmetric blister and $e = 1$ for a one-dimensional straight-edge V-peel).

4. DISCUSSION

We have shown how the blister profile changes as the film's flexural rigidity decreases. A family of curves, $\theta(\xi)$, is shown in Figure 5 for various β. These curves are all characterized by (i) a zero debonding angle at the plate edge (or delamination front) at all β; (ii) a plateau (of constant θ) at a short distance from the plate edge ($\xi = 1$) before decreasing steeply towards the plate center; (iii) a minimum, which

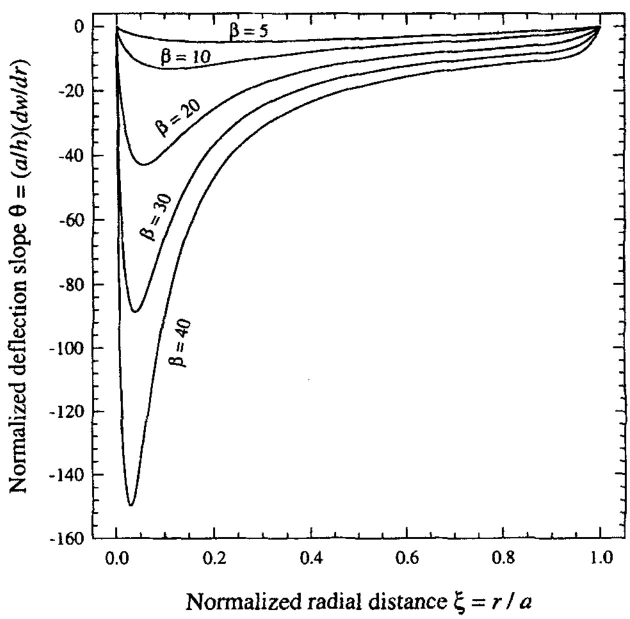

FIGURE 5 Normalized blister deflection slope for $\beta = 5$, 10, 20, 30 and 40.

shifts progressively to the plate center ($\xi = 0$) and increases in amplitude as β increases; and (iv) $\theta = 0$ at the plate center. If a steel ball of finite radius is to replace the central point load to prevent film puncture, the contact zone between the ball and the film will conform to a sphere. If this inner region covers the minimum or the "valley" of the $\theta(\xi)$ curve, the outer non-contact annulus will assume a constant θ in the plateau of $\theta(\xi)$, or a straight-sided cone [3]. Note that at an infinite β, $\theta|_{\xi=0}$ is no longer zero but approaches infinity. The gradient continuity at the plate center is lost as a consequence of the singular stress.

The contribution of bending and stretching to the effective mechanical energy release rate can be separated and analyzed on their own. It can be shown that

$$\begin{aligned} G &= G_b + G_s \\ &= \frac{M^2}{2D} + \frac{N^2(1-\nu^2)}{Eh} \end{aligned} \tag{20}$$

where the first term corresponds to bending (subscript b) and the second term stretching (subscript s). The bending moment, M, at the plate edge is found to be

$$\begin{aligned} M &= -D\left(\frac{d^2w}{dr^2}\right)_{r=a} \\ &= -\left(\frac{Dh}{a^2}\right)\theta'|_{\xi=1} \\ &= \left(\frac{Dh\varphi}{2a^2}\right)\left\{\frac{2}{\beta^2} + \frac{[I_0(\beta)+I_2(\beta)][1-\beta K_1(\beta)]}{\beta I_1(\beta)} + [K_2(\beta)-K_0(\beta)]\right\}. \end{aligned} \tag{21}$$

From (6) and Figure 5, it is obvious that $\theta'|_{\xi=1}$ diminishes as β increases. Hence M tends to a maximum of $(Dh/2a^2)\varphi$ when $\beta \approx 0$ and vanishes when $\beta \to \infty$. Substituting (21) into (20), an alternative expression can be found for χ such that

$$\begin{aligned} \chi &= \chi_b + \chi_s \\ &= \frac{\theta'^2}{4\varphi W_0} + \frac{\beta^4}{24\varphi W_0} \end{aligned} \tag{22}$$

which is equivalent to (19). The individual contributions of bending and stretching to G are plotted alongside $\chi(\beta)$ in Figure 4. In a pure bending plate, $M = F/4\pi$, $G_b = (1/2)(Fw_0/\pi a^2)$ and $\chi_b = 1/2$. Significant bending is present in the small annulus bounded by the debonding front and the θ-plateau (Fig. 5). As β increases, this annular area diminishes and eventually vanishes. In a pure stretching film, assuming a negligible contact area between the shaft end and the film, $G_s = (1/4)(Fw_0/\pi a^2)$ and $\chi_s = 1/4$. These results are identical to that derived in the previous section.

5. CONCLUSION

We have shown how the mechanical response, with or without delamination, of a thin film adhered onto a rigid substrate is formulated. The elastic solutions for a pure bending plate when both the membrane stress and blister height are small, a pure stretching membrane when both N and w_0 are large, and the intermediate are compared favorably with the classical results.

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